## CORRESPONDENCE

### WHAT IS NATURE'S ERROR CRITERION?

It is well known that the Fourier series is not the only trigonometric polynomial that may be used to represent a periodic function. It is a polynomial with the property that the mean square error between a partial sum and the given function is a minimum; that is to say, it approximates the given function so as to make the mean square error a minimum. This error criterion is only one of many that could be stipulated as fixing the manner in which the polynomial approximates the given function, and from a practical standpoint it isn't even a good one for many applications because it suffers from the Gibbs phenomenon. A Tschebyscheff-like approximation or the one inherent in the Cesaro sum which converges uniformly even at points of discontinuity may be preferable in many cases.

As we change the error criterion underlying the determination of an infinite trigonometric polynomial approximating a given periodic function, we obtain different magnitudes for the amplitudes of the fundamental and the harmonics. Hence for a given periodic function like the square wave, for example, there is no unique answer as to how large the fundamental or any given harmonic amplitude is. One cannot simply say: "How large is the third harmonic in this square wave?" In order that a unique answer may be possible, one must say: "How large is the third harmonic in this square wave assuming that such-and-such an error criterion is implied?"

Now suppose we have a square wave, and we use it to excite a simple resonant circuit tuned to the third harmonic and suppose we somehow successively reduce the losses present so that the circuit approaches infinite selectivity and assymptotically responds to the third harmonic frequency alone. What size third harmonic will it measure? Will it measure the third harmonic of a Fourier series? Will it measure the third harmonic of a Cesaro sum? Will it measure the third harmonic in a series that converges so as to yield a Tschebyscheff-like approximation? In other words: *"What is nature's error criterion?"* Everybody seems to have taken it for granted that the mean square error criterion leading to the Fourier series is also nature's error criterion, but no one has ever shown (to my knowledge, at least) that this is actually the case, much less why it is so if it *is* so.

With best wishes for a sleepless night, I remain

Faithfully yours,

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